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Visualizing Confidence Bands for Semiparametrically Estimated Nonlinear Relations Among Latent Variables

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Structural equation mixture models (SEMMs), when applied as a semiparametric model (SPM), can adequately recover potentially nonlinear latent relationships without their specification. This SPM is useful for exploratory analysis when the form of the latent regression is unknown. The purpose of this article is to help users familiar with structural equation models to add SEMM to their toolkit of exploratory analytic options. We describe how the SEMM captures potential nonlinearity between latent variables, and how confidence bands (CBs; point wise and simultaneous) for the recovered latent function are constructed and interpreted. We then illustrate the usefulness of CBs for inference with an empirical example on the effect of emotions on cognitive processing. We also introduce a visualization tool that automatically generates plots of the latent regression and their CBs to promote user accessibility. Finally, we conclude with a discussion on the use of this SPM for exploratory research.

Keywords: *exploratory; latent variables; point-wise confidence intervals; simultaneous confidence envelopes; structural equation mixture model*

Structural equation models (SEMs) are widely used in the educational, social, and behavioral sciences, as many phenomena of interest such as intelligence, emotions, and personality are latent in nature. Latent variable models have the advantage of accounting for measurement error and have the ability to model complex multivariate relationships, including latent bivariate ones. Parametric approaches of the SEM require explicit specification of the functional relationship between latent predictor and latent outcome, and are appropriate for modeling known forms (e.g., linear, quadratic, and exponential). During the exploratory phase of research, when the functional form linking latent predictor and latent outcome is unknown, using a flexible approach that can describe the latent relationship is more appropriate and practical.

Structural equation mixture models (SEMMs; Arminger & Stein, 1997; Arminger, Stein, & Wittenberg, 1999; Dolan & van der Maas, 1998; Jedidi, Jagpal, & DeSarbo, 1997a, 1997b; Muthén, 2001) can be applied as a semiparametric model (SPM) to recover latent variable relations without their explicit specification (Bauer, 2005). The mixture in this indirect application of SEMM (Titterington, Smith, & Makov, 1985) is used as a statistical expedience to estimate the global latent function where the component distributions are not taken to reflect true groups in the population. This latent variable SPM is analogous to the locally weighted scatterplot smoothing algorithm (LOWESS; Cleveland, 1979; Cleveland & Devlin, 1988) that was developed to uncover nonlinear relationships among observed variables. In LOWESS, a global regression function is estimated by smoothing over locally linear regression estimates obtained from overlapping localized subsets of the data. Similarly, the SPM recovers the global latent function by aggregating across locally linear components using the SEMM mixing probabilities as weights. More detail on this analogy is in Pek, Sterba, Kok, and Bauer (2009).

The SPM approach to SEMM is an exploratory tool that was developed to visually depict unspecified relationships between latent predictor and latent outcome (Bauer, 2005; Bauer, Baldasaro, & Gottfredson, 2012; Pek, Sterba, Kok, & Bauer, 2009). After all, "the picture-examining eye is the best finder we have of the wholly unanticipated" (Tukey, 1980, p. 24). Displays of such recovered regression functions are further enhanced by including confidence bands (CBs), which quantify and communicate the sampling variability or precision of the estimates (Wilkinson & the Task Force on Statistical Inference, 1999). Recognizing the importance of the information afforded by confidence sets (CSs), the American Educational Research Association (2006) and the American Psychological Association (2010) have urged applied researchers to report them in their publication outlets. Note that CSs refer collectively to CBs, confidence intervals (CIs), and confidence envelopes (CEs).

Confidence bands for this latent variable SPM, which include recently developed point-wise CIs (Pek, Losardo, & Bauer, 2011) and simultaneous CEs (Pek & Chalmers, 2015), also contain the same information required to conduct a null hypothesis significance test (NHST). Multiple point-wise CIs for the outcome, constructed over the range of the predictor, would form a point-wise CB; whereas multiple simultaneous CIs would form a CE or simultaneous CB. The NHST is generally unavailable for evaluating the form of the latent regression function in this SPM due to the absence of specific parameters that determine the relationship linking predictor and outcome. Because this SPM approach is mainly a graphical one, CBs offer the primary means to statistically evaluate the effect of latent predictor on latent outcome.

For the purpose of adding SEMM to researchers' toolkit of exploratory analytic options, this article provides a demonstration of the relatively less known indirect approach of SEMM, formulated as an SPM, that can recover unspecified relationships between latent predictor and latent outcome. Although our focus is on the SPM, we first review the conventional linear SEM and provide a nontechnical overview of point-wise CIs and simultaneous CEs for pedagogical purposes. Next, the linear SEM is extended to the SPM approach that employs SEMM. The construction and interpretation of CIs and CEs for this SPM is also briefly explicated. An empirical example examining the relationship between emotions and cognitive processing is used to illustrate this SPM approach. Finally, to promote end user accessibility to this relatively complex set of exploratory modeling tools, we introduce a freely available visualization tool that automatically generates plots of the latent variable regression along with the two types of CBs.

Parametric Linear Latent Regression

Linear relationships between latent variables are readily modeled with the linear SEM. For simplicity, we present model equations for one latent predictor η_1 and one latent outcome η_2, although the following developments may be extended to include latent bivariate relationships nested within more complex models.

In the linear SEM, the measurement models for the latent variables are given by:

$$\begin{aligned} \mathbf{y}_{1i} &= \boldsymbol{\nu}_1 + \boldsymbol{\lambda}_1\eta_{1i} + \boldsymbol{\varepsilon}_{1i} \\ \mathbf{y}_{2i} &= \boldsymbol{\nu}_2 + \boldsymbol{\lambda}_2\eta_{2i} + \boldsymbol{\varepsilon}_{2i}, \end{aligned} \tag{1}$$

where $\mathbf{y}_1$ and $\mathbf{y}_2$ are vectors of observed variables measuring the latent predictor and latent outcome for individual i. We make use of the LISREL all-y notation, where the subscript 1 denotes the x-side of the model, and the subscript 2 denotes the y-side of the model. The intercepts and slopes (or loadings) for the regression of the observed variables on the latent factors are expressed in the vectors $\boldsymbol{\nu}$ and λ, respectively. The residuals are represented by $\boldsymbol{\varepsilon}$ and have a joint zero mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Theta}$. To reflect the assumption that the observed variables $\mathbf{y}$ are locally independent, after conditioning on the latent variables η, $\boldsymbol{\Theta}$ is typically constrained to be a diagonal matrix.

The latent linear regression model is given by:

$$\begin{aligned} \eta_{1i} &= \alpha_1 + \zeta_{1i} \\ \eta_{2i} &= \alpha_2 + \beta_{21}\eta_{1i} + \zeta_{2i}. \end{aligned} \tag{2}$$

The latent predictor η_1 has mean α_1 and variance $\mathrm{VAR}(\zeta_1) = \psi_{11}$, and the latent outcome η_2 has intercept α_2, slope β_{21}, and residual variance $\mathrm{VAR}(\zeta_2) = \psi_{22}$. It is assumed that ζ_1 and ζ_2 are independent of each other and of $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$. Specific mean and covariance structures for the vector of observed variables $\mathbf{y}$ are implied by Equations 1 and 2 and are denoted by $\mu(\boldsymbol{\theta})$ and $\boldsymbol{\Sigma}(\boldsymbol{\theta})$, respectively, with $\boldsymbol{\theta}$ representing the vector of model parameters (Bollen, 1989). Given the assumption that all residuals are multivariate normally distributed, the joint marginal probability distribution function (PDF) for the modeled data is multivariate normal $\phi[\mathbf{y}; \mu(\boldsymbol{\theta}), \boldsymbol{\Sigma}(\boldsymbol{\theta})]$, where $\phi[\cdot]$ denotes the multivariate normal PDF. This PDF provides the basis for maximum likelihood (ML) estimation of the parameters. By computing the expected value of the latent outcome η_2 in Equation 2, estimates of the latent linear regression may be obtained from the following expression:

$$E[\eta_2|\eta_1] = \alpha_2 + \beta_{21}\eta_1, \tag{3}$$

where $E[\cdot]$ is the expectation operator. Here, the specified linear form of the latent regression is solely determined by the two parameters α_2 and β_{21}.

Exact Confidence Bands

Two kinds of CBs, point-wise or nonsimultaneous CIs and simultaneous CEs, may be constructed around any regression function. For the parametric linear SEM in Equation 3, exact Wald-type CBs (cf. Wald, 1943) are typically constructed using the familiar equation: estimate $\pm$ critical value $\times$ standard error of estimate:

$$\hat{E}[\eta_2|\eta_1] \pm \chi_{p,1-\alpha}[\widehat{\mathrm{VAR}}(\hat{E}[\eta_2|\eta_1])]^{1/2}. \tag{4}$$

Assuming asymptotic normality of the sampling distribution of $\hat{E}[\eta_2|\eta_1]$, the critical value is $\chi_{p,1-\alpha}$ or the $(1-\alpha)$th quantile of the square root of the χ^2 distribution with p degrees of freedom. Note that $z = \chi_1$, and the standard normal distribution is the square root of the χ^2 distribution with $p = 1$ degrees of freedom (e.g., see Leemis & McQueston, 2008). As the latent linear regression function is additive in its parameters, computation of the standard error of estimate of Equation 3 is exact and given by:

$$[\widehat{\mathrm{VAR}}(\hat{E}[\eta_2|\eta_1])]^{1/2} = [\widehat{\mathrm{VAR}}(\hat{\alpha}_2) + 2\eta_1\widehat{\mathrm{COV}}(\hat{\alpha}_2, \hat{\beta}_{21}) + \eta_1^2\widehat{\mathrm{VAR}}(\hat{\beta}_{21})]^{1/2}, \tag{5}$$

where $\widehat{\mathrm{VAR}}(\cdot)$ is the estimate of the variance of the relevant estimated parameter, and $\widehat{\mathrm{COV}}(\hat{\alpha}_2, \hat{\beta}_{21})$ is the estimate of the covariance between the intercept and the slope estimates. Wald-type CIs and CEs differ in critical values used in their computation. Note that CIs and CEs are distinct from prediction intervals and prediction envelopes. The former pair is constructed about the mean outcome

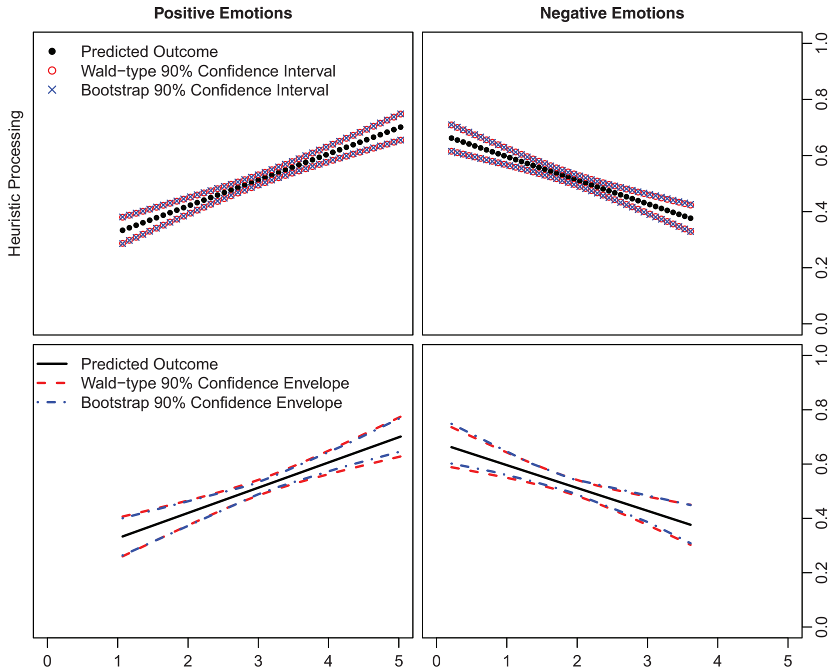

FIGURE 1. *Wald-type and parametric bootstrap confidence intervals and confidence envelopes for the linear effects of positive and negative emotions on heuristic processing.*

or mean regression function, whereas the latter pair is constructed about a new observation or a regression function for new observations.

Confidence intervals. Point-wise CIs for the linear latent regression are constructed using critical values based on the $(1-\alpha)$th quantile of the square root of the χ^2 distribution with $p = 1$ degrees of freedom $\chi_{1,1-\alpha}$, or the more familiar $\alpha/2$th quantile of the standard normal distribution $z_{1-\alpha/2}$. For a 95% CI, $\chi_{1,.95} = z_{.975} = 1.96$. Suppose that $\hat{E}[\eta_2|\eta_1 = 10] = 1.2$, with 95% CI [1.0, 1.4]. Over repeated sampling, such a CI is expected to capture $E[\eta_2|\eta_1 = 10]$ 95% of the time. Recall that CSs also contain the necessary information required to conduct an NHST. Suppose that $H_0 : E[\eta_2|\eta_1 = 10] = 0$ is to be evaluated. Because the 95% CI does not contain the null value of zero, H_0 is rejected at $p < .05$, and $E[\eta_2|\eta_1 = 10]$ is significantly different from zero.

Point-wise CIs are based on an α level or Type I error rate that is valid for a *single* population value. Such CIs were not developed for inference about multiple population values let alone an infinite number of population values that represent a regression function as a whole. Although multiple point-wise CIs could be constructed about a regression function to form a point-wise CB (e.g., see top row of plots in Figure 1), the Type I error rate associated with the set of multiple CIs will accumulate or become inflated. Consider the simple instance of two estimates $\hat{E}[\eta_2|\eta_1 = 10] = 1.2$ and $\hat{E}[\eta_2|\eta_1 = 15] = 2.0$ with 95% CIs [1.0, 1.4] and [1.7, 2.3], respectively. Over repeated sampling, such CIs would independently contain $E[\eta_2|\eta_1 = 10]$ and $E[\eta_2|\eta_1 = 15]$ 95% of the time. The coverage rate of .95 pertains to each interval estimate, independent of the other. When both population values are considered simultaneously, statistical theory assures that the joint coverage of these two CIs for $E[\eta_2|\eta_1 = 10]$ and $E[\eta_2|\eta_1 = 15]$ would be less than or equal to 95%.

Reduction in the coverage of point-wise CIs for multiple population values is better known as the issue of multiplicity or inflation of the family-wise Type I error under multiple testing. Each of the two CIs in the running example does not capture their relevant null hypothesis, that is, $H_0 : E[\eta_2|\eta_1 = 10] = 0$ and $E[\eta_2|\eta_1 = 15] = 0$ can be rejected *independently* of each other, at $p < .05$. In contrast, the joint null hypothesis $H_0 : E[\eta_2|\eta_1 = 10] = 0 \bigcap E[\eta_2|\eta_1 = 15] = 0$ may or may not be rejected at $p < .05$. Here, $\cap$ is the intersection between the two hypotheses, requiring both conditions to occur jointly. When multiple points are tested simultaneously, the family-wise Type I error rate $\alpha_{fw} \geq .05$. Adjustments such as the Bonferroni approach can be applied to multiple point-wise CIs to allow for joint inference by controlling the family-wise α_{fw} level (e.g., see Neter, Kutner, Nachtsheim, & Wasserman, 1996, pp. 157–158). Yet, such post hoc corrections are not well suited to evaluate large numbers of estimated values, such as an infinite set representing the regression function as a whole. The Bonferroni method has been shown to be overly conservative in such contexts (Perneger, 1998). Instead, CEs or simultaneous CBs should be constructed when inference about the regression function as a whole is of interest.

Confidence envelopes. The extension of the CI for a single point to a CS for a regression function was achieved in the seminal work of Working and Hotelling (1929). They called their development a CE, which envelopes the regression function as a whole. Wald-type simultaneous CEs for the latent linear regression are constructed with critical values based on the $(1 - \alpha)$th quantile of the square root of the χ^2 distribution with $p = 2$ degrees of freedom, $\chi_{2,1-\alpha}$. Here, $p = 2$ because two parameters define the regression function in Equation 3, and for a 95% CE, $\chi_{2,.95} = 2.45$. The critical value is determined by a Scheffé (1953) correction, which rests on the sampling distribution of χ^2_{maximum} where all possible values of the latent predictor may be tested while maintaining the family-wise error rate (Maxwell & Delaney, 2004). Point-wise CIs are therefore smaller or equal to the commensurate slices of the CE, at conditional values of η_2, as $\chi_{1,1-\alpha} \leq \chi_{p,1-\alpha}$. Additionally, in contrast to point-wise CIs that communicate the precision of a single estimate that lies on the regression function, CEs communicate the precision of the infinite number of estimates that make up the entire regression function.

Suppose that the following estimates $\hat{\alpha}_1 = 2.4$ and $\hat{\beta}_{21} = -0.7$ were obtained for the latent linear regression of Equation 3, and a 95% CE was constructed. Over repeated sampling, the population latent linear regression function is expected to fall within such an envelope 95% of the time. Note that the CE is typically communicated visually, as it is composed of an infinite number of points across the latent predictor η_1 (e.g., see the bottom row plots of Figure 1). Given the relationship between CBs and NHSTs, CEs also allow for inference about the regression function as a whole. Consider the null hypothesis $H_0 : E[\eta_2|\eta_1] = 3.0 - 1.5\eta_1$. If the 95% CE for the linear function does not contain or intersect the null function, H_0 is rejected at $p < .05$, and the population function is significantly different from $E[\eta_2|\eta_1] = 3.0 - 1.5\eta_1$.

Recall that the linear latent regression of Equation 3 is a parametric model, and the form linking latent predictor with latent outcome is specified by the intercept (α_2) and slope (β_{21}) parameters. Instead of using the CE to evaluate $H_0 : E[\eta_2|\eta_1] = 3.0 - 1.5\eta_1$, an NHST may instead be set up to test the equivalent null hypothesis $H_0 : \alpha_1 = 3.0 \bigcap \beta_{21} = -1.5$. Note the one-to-one mapping between the parameter space for $\boldsymbol{\theta}_p$ and the functional space $E[\eta_2|\eta_1]$ by $g(\boldsymbol{\theta}_p) = E[\eta_2|\eta_1]$. Another advantage of CEs over NHST, beyond providing estimate precision, is that CEs afford analysts the flexibility of testing different functional forms between predictor and outcome. Continuing with the working example, the constructed CE may be used to test different functional forms, such as $H_0 : E[\eta_2|\eta_1] = 3.0 - 1.5\eta_1 + .1\eta_1^2$ or $H_0 : E[\eta_2|\eta_1] = 3.5 - e^{.5\eta_1}$, where the parameters in the null hypotheses need not correspond to those that are estimated from the data. This property of CEs is especially useful as an inferential device when paired with a semiparametric method that can recover the unspecified form of the relationship between latent predictor and latent outcome that is described below.

Flexible Semiparametric Latent Regression

Latent bivariate relationships can be modeled without explicit specification of their form by applying SEMMs as an SPM. This SPM is an extension of the linear SEM defined in Equations 1 and 2. Unlike linear SEM, where the joint distribution of the vector of observed variables in **y** is taken to be multivariate normal, the SEMM here assumes that the joint distribution of **y** can be approximated by $k = 1, \ldots, K$ multivariate normal distributions. The latent variable model is then given by:

$$\begin{aligned} \eta_{1i[k]} &= \alpha_{1[k]} + \zeta_{1i[k]} \\ \eta_{2i[k]} &= \alpha_{2[k]} + \beta_{21[k]}\eta_{1i[k]} + \zeta_{2i[k]}. \end{aligned} \tag{6}$$

Compared to Equation 2 of the linear SEM, Equation 6 of the SPM has additional subscripts k associated with each parameter that allows for different means $\alpha_{1[k]}$, intercepts $\alpha_{2[k]}$, slopes $\beta_{21[k]}$, variances of the latent predictor $\mathrm{VAR}(\zeta_{1[k]}) =$

$\psi_{11[k]}$, and residual variables of the latent outcome $\mathrm{VAR}(\zeta_{2[k]}) = \psi_{22[k]}$ for each component or class k. Some or all of the parameters in Equation 7 are required to differ across the K latent classes for the model to flexibly recover potentially nonlinear latent relationships, although $\psi_{11[k]}$ and $\psi_{22[k]}$ could be optionally constrained to be equal over classes. In practice, model selection indices are typically used to determine the constraints placed on $\psi_{11[k]}$ and $\psi_{11[k]}$. The linear SEM within each k class is assumed to have the measurement model of Equation 1, implying that the measurement model is invariant over the K classes. This measurement invariance constraint ensures that the latent variables are equivalently defined for all individuals in the population (Meredith, 1993).

Let $P(k)$ denote the mixing probability for each latent Class k such that $\sum_{k=1}^{K} P(k) = 1$. The expected value of the latent outcome within Class k as defined in Equation 7, which is analogous to Equation 3, is:

$$E_{[k]}[\eta_2|\eta_1] = \alpha_{2[k]} + \beta_{21[k]}\eta_{1[k]}. \tag{7}$$

These K within-class or local relationships between the latent variables are linear, and the flexible global relationship between the latent variables is obtained by taking the expected value across the K components. The conditional mixing probabilities $P(k|\eta_1)$ are then used as smoothing weights to aggregate across the locally linear relationships in Equation 7:

$$E[\eta_2|\eta_1] = \sum_{k=1}^{K} P(k|\eta_1)E_{[k]}[\eta_2|\eta_1], \tag{8}$$

where

$$P(k|\eta_1) = \frac{P(k)\phi_{[k]}[\eta_1; \alpha_{1[k]}, \psi_{11[k]}]}{\sum_{k=1}^{K} P(k)\phi_{[k]}[\eta_1; \alpha_{1[k]}, \psi_{11[k]}]} \tag{9}$$

is the conditional probability of class membership at a given value of the latent predictor. For simulated and empirical examples showing how the method works to recover latent relationships, see Pek et al. (2009), and for technical details, see Bauer (2005).

From a series of targeted simulations, this SPM has been shown to adequately recover different types of nonlinear latent functions with limited bias (Bauer, 2005; Bauer et al., 2012; Pek & Chalmers, 2015; Pek et al., 2011). Bias of the aggregate function is reduced by selecting more mixing components at the expense of estimate efficiency. In general, two other factors influence bias. First, bias tends to be larger at regions of high curvature which the linear local function tends to inadequately approximate. Second, bias also tends to be large at the tail ends of the latent predictor where data are sparse. The number of mixing components is often determined by the Akaike Information Criterion (AIC; Akaike, 1974) or the Bayesian Information Criterion (BIC; G. Schwarz, 1978), with the AIC tending to select more classes than the BIC. Simulations have demonstrated that models with minimum AIC estimate the form of the unknown function with less bias and higher sampling variability compared to models with minimum BIC (Bauer et al., 2012).

Unlike parametric latent regression models (e.g., the linear latent regression), the parameters in the flexible latent regression serve as a means to recover the unknown latent function and have no meaningful interpretation. As the parameters of the SPM in Equation 8 are nuisance parameters, NHSTs are not directly available for testing certain research questions such as linearity of the latent relationship. Instead, CBs become the primary inferential device for statistically evaluating the form of the flexible latent regression.

Approximate Confidence Bands

Like the linear SEM, two kinds of CBs may be constructed about the flexible latent regression. Recently developed approaches for constructing CBs for this flexible latent regression are not exact due to the nonlinear function of the conditional mixing probabilities in Equation 9. These developments provide two methods to construct approximate CIs (Pek et al., 2011) and CEs (Pek & Chalmers, 2015). The first method is the familiar Wald-type CBs, and the second method is the parametric bootstrap CBs.

Wald-type confidence bands. Wald-type CBs for the flexible latent regression have been generally defined in Equation 4. As $\hat{E}[\eta_2|\eta_1]$ is nonlinear in its parameters for the SPM, the standard error of estimate is obtained via the delta method. In brief, the delta method linearizes the expression of $E[\eta_2|\eta_1]$ in Equation 8 with a first-order Taylor Series expansion, resulting in an approximate standard error of estimate. Raykov and Marcoulides (2004) provide a tutorial on applying the method in SEMs. Formulas underlying the delta method standard error of estimate for the SPM are provided in Pek, Losardo, and Bauer (2011).

Recall that Wald-type CIs and CEs differ only in the critical values used in Equation 4. Point-wise CIs are computed with $p = 1$ degrees of freedom, whereas simultaneous CEs are constructed with p degrees of freedom (Scheffé, 1953; Working & Hotelling, 1929). Specifically, the p degrees of freedom are the number of parameters present in the latent regression expressed in Equation 8. For instance, $p = 9$ for a $K = 2$ class solution with two intercepts ($\alpha_{2[1]}$ and $\alpha_{2[2]}$), two slopes ($\beta_{21[1]}$ and $\beta_{21[2]}$), two means ($\alpha_{1[1]}$ and $\alpha_{1[2]}$), two variances ($\psi_{11[1]}$ and $\psi_{11[2]}$), and one class probability ($P(k = 1)$). Only one class probability is required to define two class probabilities because $P(k = 2) = 1 - P(k = 1)$.

For Wald-type CBs to have proper coverage, it is assumed that the sampling distribution of $\hat{E}[\eta_2|\eta_1]$ is asymptotically normally distributed. Coverage is the probability of how often the CBs capture the population value(s) over repeated sampling. This assumption allows the use of $\chi_{p,1-\alpha}$ as the critical value. However, there may be instances, such as limited sample size, where the sampling distribution of the global regression is non-normal. In such cases, bootstrap CBs may be a reasonable alternative to Wald-type CBs.

Parametric bootstrap confidence bands. The bootstrap is an empirically based resampling algorithm that is typically employed to estimate the sampling distribution of an estimate (see Efron & Tibshirani, 1993, for a good introduction). The conventional nonparametric bootstrap approach is relatively untenable for the computationally burdensome SPM, and parametric bootstrap CBs are constructed instead (Pek & Chalmers, 2015; Pek et al., 2011). Rather than drawing bootstrap replicates from the data in the nonparametric approach, bootstrap replicates are drawn from the model for the data defined in Equation 8 under the parametric approach. Given the duality or one-to-one mapping between the parameter space $\boldsymbol{\theta}_p$ and function space $E[\eta_2|\eta_1]$, and the asymptotic normality of ML estimates, bootstrap replicates are drawn from $\phi\left[\mu(\hat{\boldsymbol{\theta}}_p), \boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_p)\right]$. Consider the case where $K = 1$, such that Equation 8 reduces to Equation 3, and a simple latent linear regression is obtained. Parametric bootstrap CBs for this model are based on bootstrap replicates drawn from the multivariate normal distribution $\phi\left[\begin{pmatrix}\hat{\alpha}_2 \\ \hat{\beta}_{21}\end{pmatrix}, \begin{pmatrix}\widehat{\mathrm{VAR}}(\hat{\alpha}_2) & \widehat{\mathrm{COV}}(\hat{\alpha}_2, \hat{\beta}_{21}) \\ \widehat{\mathrm{COV}}(\hat{\beta}_{21}, \hat{\alpha}_2) & \widehat{\mathrm{VAR}}(\hat{\beta}_{21})\end{pmatrix}\right]$.

To construct point-wise CIs, an arbitrarily large number of B_{CI} replicates are randomly drawn from the parametric model or multivariate normal distribution of the p ML parameter estimates to obtain B_{CI} sets of parameter estimates. For instance, with $B_{CI} = 1{,}000$ sets of parameter estimates, 1,000 different estimated latent regression functions may be computed across the range of the latent predictor η_1. At conditional values of η_1, lower and upper point-wise CIs are defined as the $[(\alpha/2)100\%\ B_{CI}]$th and $[(1-\alpha/2)100\%\ B_{CI}]$ ordered value among the set of bootstrapped latent regression values. With the example of $B_{CI} = 1{,}000$, the lower and upper bounds of the 95% CI are the 25th and 975th ordered bootstrapped value at a given latent predictor value η_1.

In a similar fashion, simultaneous parametric bootstrap CEs are constructed with the B_{CE} bootstrap replicates empirically estimating the sampling distribution of the aggregate function $\hat{E}[\eta_2|\eta_1]$ as a whole. Unlike the approach of sorting bootstrap estimates at each value of η_1 to obtain upper and lower bounds of the CI, bounds of the CE are defined as the boundary of the overlap of a predetermined number of B_{CE} bootstrap regression functions (Pek & Chalmers, 2015). The number of B_{CE} replicates is determined by the p number of parameters in Equation 8 and the error rate α. In particular, B_{CE} is based on the expected value of the range of $\chi_{p,1-\alpha}[\widehat{\mathrm{VAR}}(\hat{\boldsymbol{\theta}}_p)]^{1/2}$ (see online appendix of Pek and Chalmers, 2015, for technical details and a simple example). For the linear latent regression example, where $p = 2$, $B_{CE} \approx 85$, and the boundary of these 85 bootstrapped regression lines form the CE (cf. Thissen & Wainer, 1990). The two plots in the bottom row of Figure 1 illustrate Wald-type and parametric bootstrap CIs and CEs for the simple linear regression with $p = 2$.

Properties of the Approximate Confidence Bands

Several important similarities and distinctions between the different types of CBs for the SPM should be noted for practical applications. The properties of Wald-type and parametric bootstrap CBs are first highlighted, followed by a review of the different uses of CIs and CEs in the context of applying the SPM in exploratory analyses.

Wald-type versus bootstrap confidence bands. Both approaches to generating CBs for the SPM require asymptotic normality of the set of parameter estimates $\hat{\boldsymbol{\theta}}_p$. The Wald-type approach is an analytical method that assumes asymptotic normality of the sampling distribution of $g(\hat{\boldsymbol{\theta}}_p) = \hat{E}[\eta_2|\eta_1]$, leading to symmetric CBs as defined in Equation 4. In contrast, the parametric bootstrap method assumes asymptotic normality of the parameter estimates $\phi\left[\mu(\hat{\boldsymbol{\theta}}_p), \boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_p)\right]$ that serves as a model with which bootstrap replicates are drawn. The sampling distribution of $\hat{E}[\eta_2|\eta_1]$ is estimated by the bootstrap replicates, and typically results in asymmetric parametric bootstrap CBs. Calculating Wald-type CBs are immediate as they are defined by closed-form solutions, whereas estimating the empirical parametric bootstrap CBs tends to be computationally intensive.

Recall that these two methods generate approximate CBs about the aggregate latent regression function. For the Wald approach, the delta method is employed to approximate the nonlinear Equation 8 with a linear function to simplify computations. Similarly, the parametric bootstrap approach does not result in exact CBs because a model is used as an approximation to the data in contrast to the nonparametric bootstrap approach. The approximations underlying both methods to constructing CBs improve with larger sample size. Monte Carlo studies show that the parametric bootstrap CI has better coverage at lower sample sizes, and the Wald-type CI has better coverage at larger sample sizes across symmetric and asymmetric population functions (Pek et al., 2011). Sample size had little effect on the coverage of Wald-type and parametric bootstrap CEs (Pek & Chalmers, 2015).

In general, these simulation studies confirm that Wald-type CBs tend to be more conservative compared to parametric bootstrap CBs. Excluding tail-end values of the latent predictor η_1, where data are sparse, coverage for the parametric bootstrap CI was at the nominal $(1 - \alpha)\%$. In contrast, coverage for the Wald-type CI tended to exceed the nominal $(1 - \alpha)\%$. The better coverage of the parametric bootstrap CIs came at the cost of tracking bias more closely due to its empirical nature. Specifically, coverage of the bootstrap CIs was liberal at specific ranges of the latent predictor when bias was present. In contrast,

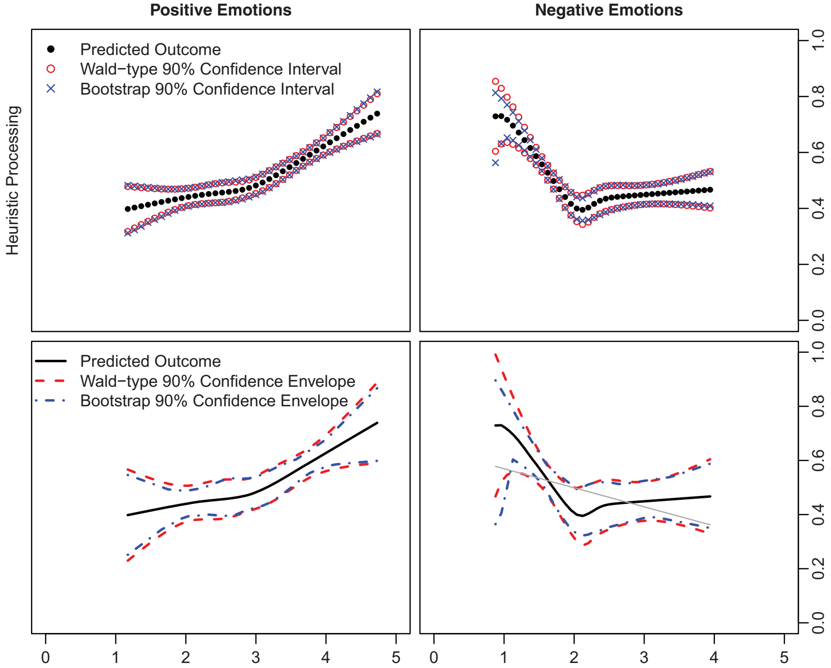

FIGURE 2. *Wald-type and parametric bootstrap confidence intervals and confidence envelopes for the recovered effect of positive and negative emotions on heuristic processing. The fine gray solid line within the 90% Wald-type confidence interval suggests that the latent function could be linear.*

Wald-type CIs tended to be robust against bias and had larger than nominal coverage rates. For the CEs, the parametric bootstrap approach had very slightly better coverage rates than the Wald-type method. Despite these noted differences in the performance of Wald-type and parametric bootstrap CBs, these approximate CBs were observed to be highly similar for a single data set (see Figures 1 and 2).

Confidence intervals versus confidence envelopes. Different types of research questions are answered with CIs and CEs. When the effect of a single latent predictor value is of interest (e.g., a meaningful threshold), CIs are employed. CBs formed by multiple point-wise CIs are discouraged, as the CB would convey an overly liberal confidence level about the regression function. Suppose that the relationship between motivation (η_1) and math ability (η_2) is explored, and a motivation score of $\eta_1 = 5$ is at a level ideal for intervention, and a math ability score of $\eta_2 = 30$ indicates competence. A 95% CI for $E[\eta_2|\eta_1 = 5]$ provides a range of plausible population math ability scores associated with the given motivation score. Additionally, the CI allows for conducting an NHST of $H_0 : E[\eta_2|\eta_1 = 5] = 30$. Results of such an NHST could indicate that at this low level of motivation, which is associated with intervention efficacy, math ability scores are below an acceptable threshold. This CI and its associated NHST could thus inform of the viability of implementing an intervention to boost low motivation in order to improve math ability scores. During the exploratory stage of research, when a specific level of η_1 in relation to η_2 is examined, CIs communicate estimate precision and can be inverted to conduct NHSTs when such tests are meaningful.

In contrast, CEs are used when the form of the regression as a whole is of interest. Recall that in the SPM, the model parameters are used as an expedience to recover the unknown function and no single parameter can be tested to evaluate the effect of the latent predictor on the outcome. In contrast, suppose a parametric model with $E[\eta_2|\eta_1] = \alpha_2 + \beta_{21}\eta_1 + \beta_{31}\eta_1^2$ is specified. Here, it may be concluded that the effect of η_1 on η_2 is nonlinear when an NHST of $H_0 : \beta_{31} = 0$ is rejected. Nonlinearity of the latent function that is recovered via the SPM is, instead, informed by CEs. In particular, an NHST of $H_0 : E[\eta_2|\eta_1] = \alpha_0 + \beta_0\eta_1$ is set up such that its rejection would indicate nonlinearity of the relationship between η_1 and η_2. Note that α_0 and β_0 can take on any real value and represent all possible linear functions. When the CE does not contain H_0, linearity is rejected (Pek & Chalmers, 2015). In practice, this test for nonlinearity is implemented as a line finding algorithm. When a line cannot be found within the CE, H_0 is rejected and nonlinearity is concluded; when a line is found within the CE, linearity cannot be rejected. Therefore, when the form of a latent relationship is unknown during exploratory analyses, use of the SPM in conjunction with CEs allows for researchers to rule out linearity.

The next section provides an illustration of Wald-type and parametric bootstrap CBs, where the effect of emotions on cognitive heuristic processing is explored. We first assume a linear association between emotions and heuristic processing as a special case of the SPM and demonstrate the use of CIs and CEs. The assumption of linearity is then relaxed, and the SPM is applied to recover the unknown form of the latent function between emotions and heuristic processing. Similarly, CIs and CEs are constructed and interpreted.

Empirical Demonstration

Research has shown that emotions reliably influence individuals' tendency to rely on heuristic processing in decision making. Individuals in a positive mood are more likely to adopt a heuristic processing strategy by relying on general knowledge structures instead of giving attention to detail (see N. Schwarz, 2000, for a review). Individuals in a negative mood, in contrast, are more likely to adopt a strategy characterized by systematic cognition and attention to detail (N. Schwarz & Clore, 1996).

A real data example ($N = 507$) is used to explore the effect of positive and negative emotions on cognitive heuristic processing. Based on past research, it is expected that positive and negative emotions are monotonically predictive of heuristic processing in opposite directions. Specifically, positive emotions are anticipated to be positively associated with heuristic processing, whereas negative emotions are hypothesized to be negatively related with heuristic processing. Details regarding the sample, operationalization of the constructs, and parameter estimates are reported in Pek et al. (2009). Model estimation was carried out with Mplus 7.1 (Muthén & Muthén, 2011). Note that theory does not inform of the functional form between emotions and heuristic processing aside from the expectation of monotonicity, that is, the effect of emotions on heuristic processing is either strictly increasing or strictly decreasing. We first make the simplifying assumption that the relationship between emotions and heuristic processing is linear and fit a $K = 1$ class SEMM model to the data. Additionally, a more liberal $\alpha = .10$ rate is used for this exploratory analysis.

Linear Latent Variable Regression

The linear form linking positive and negative emotions to heuristic processing is presented in Figure 1. Plots on the left display the linear relationship between positive emotions and heuristic processing and plots on the right display the association of negative emotions with heuristic processing. Confirming observations of previous studies, positive emotions were positively associated with cognitive heuristic processing and negative emotions were negatively related with cognitive heuristic processing.

Confidence intervals. A few general observations may be gleaned from the first row of plots in Figure 1, where each solid dot is the estimated latent outcome of heuristic processing $\left(\hat{E}[\eta_2|\eta_1]\right)$ at conditional values of emotions (η_1). The open circles and crosses depict point-wise 90% Wald-type and parametric bootstrap CIs, respectively. Confidence intervals for a linear function follow a quadratic form. Confidence intervals with the smallest widths are always located at the mean value of the latent predictor η_1 such that the most precise estimates are where the data are least sparse. Conversely, CIs indicating the poorest estimate precision are at the end points of η_1 where data are sparse. Additionally, the open circles and crosses in Figure 1 mostly fall on top of each other at each conditional latent predictor value, indicating that it makes little difference for applied researchers to interpret one type of CI over the other in this example.

Point-wise CIs allow for inference at conditional values of the latent predictor η_1, and the set of CIs that form the CB should not be used to draw inferences about the regression function. Suppose that past studies have indicated that a value of $\eta_1 = 2.5$ on positive and negative emotions is substantively important, and a minimal value of $\eta_2 = 0.5$ on heuristic processing is associated with an unfavorable decision-making style. Given a value of $\eta_1 = 2.5$ for positive emotions, the 90% Wald-type and parametric bootstrap CIs for heuristic processing are [0.49, 0.53] and [0.50, 0.53], respectively. For $\eta_1 = 2.5$ on negative emotions, the 90% Wald-type and parametric bootstrap CIs for predicted heuristic processing are both [0.39, 0.46]. A test of $H_0 : E[\eta_2|\eta_1 = 2.5] = 0.5$ for negative emotions is significant, as the null value is not contained within the CIs, implying that a value of 2.5 in negative emotions is related to an unfavorable level of heuristic processing. However, the same test is not rejected for positive emotions. The data does not support the claim that a value of 2.5 on positive emotions is associated with an undesirable level of heuristic processing.

Confidence envelopes. Wald-type and parametric bootstrap CEs for the linear association of positive and negative emotions with heuristic processing are presented in the bottom row of plots in Figure 1. The linear relationships are represented by the solid line, and the 90% Wald-type and parametric bootstrap CEs are represented by dashed and dot-dashed lines, respectively. Similar to point-wise CIs, the two types of CEs show much overlap and communicate highest precision at the centroid of the linear latent function. As CEs communicate the sampling variability of the regression function as a whole, the CEs are wider than their commensurate CIs at conditional values of the latent predictor η_1. These CEs could also be employed to test specific hypotheses about the functional relationship between emotions and heuristic processing (e.g., $H_0 : E[\eta_2|\eta_1] = \alpha_0$ for an intercept-only model, where α_0 can take on any real value). However, it is more common to conduct an equivalent NHST involving the parameters directly for this parametric model (e.g., $H_0 : \beta_{21} = 0$).

The main advantage of employing CEs for inference is when the form of the latent relationship is unknown and the SPM is used as an exploratory model. As theory does not suggest how emotions and cognitive decision making are related, it is preferable to estimate the latent regression function without a priori specification of its form. In the next section, we demonstrate the use of the SPM as an exploratory device, in conjunction with CIs and CEs, to flexibly describe and evaluate unspecified latent variable relationships.

Flexible Latent Variable Regression

The results from the linear SEM confirm that positive and negative emotions are related to heuristic processing positively and negatively, respectively. The assumption of linearity is now relaxed, and the SEMM is applied as an SPM to recover potential nonlinearity between emotions and cognitive decision making. For both emotions, the BIC indicated that $K = 2$ classes fit the data best (see Pek et al., 2009). Simulation work recommends the BIC among other information criteria to select classes in finite mixtures (Nylund, Asparouhov, & Muthén, 2007). Under the SPM, the number of classes determined by minimum BIC tends to be efficient compared to the AIC (Bauer et al., 2012) and strikes a balance between the flexibility of the SPM in capturing the unknown latent function (more bias) and sampling variability (more estimate precision). The potentially nonlinear latent regressions of heuristic processing on positive and negative emotions are presented in Figure 2. Plots on the left of the 2 × 2 array show the relationship between positive emotions and heuristic processing, and those on the right show the relationship between negative emotions on heuristic processing. The first row of plots display CIs and the second row of plots present CEs.

Consistent with theory, positive emotions were monotonically and positively associated with heuristic processing. Beyond a positive emotion value of $\eta_1 = 3.0$, the relationship between positive emotions and heuristic processing becomes more acute. This pattern suggests that heuristic processing strategies increase more sharply under the influence of a strongly positive mood. Negative emotions were found to be monotonically negatively related with heuristic processing.[1] In particular, the tendency to rely on heuristic processing decreases sharply and reaches an asymptote when negative emotions exceed a value of about $\eta_1 = 2.0$. These descriptive results from the SPM suggest that the form of the relationship of the two types of emotions to heuristic processing are distinct. The influence of negative emotions on heuristic processing is different from the absence of positive emotions, suggesting a limit to depressive realism in that individuals appear to retain some heuristic processing even under the influence of strong negative emotions (Alloy & Abramson, 1979).

Confidence intervals. In the top row of plots in Figure 2, the solid dots represent predicted values on heuristic processing at given levels of positive or negative emotions. Additionally, the open circles and crosses represent 90% Wald-type and parametric bootstrap CIs, respectively. The CIs here no longer form a point-wise CB that is quadratic in form when the SPM is used to recover the unknown form between the latent variables. As the global latent regression is an aggregation of $K = 2$ linear components, the mean levels of the latent predictor are not associated with the best estimate precision. Instead, the tightest CIs are associated with values of about $\eta_1 = 3.5$ for positive emotions and $\eta_1 = 1.5$ for negative emotions.

Comparing the first row of plots between Figures 1 and 2, the CIs depict more estimate uncertainty for the SPM compared to the linear SEM as expected; under the linear SEM, $p = 2$ parameters define the form of the latent regression, whereas $p = 9$ are used by the SPM to recover the latent relationship. Additionally, the Wald-type and parametric bootstrap CIs for the SPM show less overlap due to the approximate nature of these CIs.

As with the linear SEM, these approximate CIs can be inverted to test null hypotheses. Suppose we examine the same null hypothesis of $E[\eta_2|\eta_1 = 2.5] = 0.5$, which indicates an unfavorable level of heuristic processing when emotions have a value of 2.5. The 90% Wald-type and parametric bootstrap CIs for heuristic processing, which is associated with a value of $\eta_1 = 2.5$ on positive emotions, are both [0.45, 0.51]. For negative emotions, the 90% Wald-type and parametric bootstrap CIs for predicted levels of heuristic processing are both [0.42, 0.48]. Similar to results of the linear SEM, this null hypothesis is independently rejected for negative, and not positive emotions.

Confidence envelopes. The bottom row of plots in Figure 2 depicts 90% Wald-type and bootstrap CEs in dashed and dot-dashed lines, respectively. The potentially nonlinear effects of emotions on heuristic processing are represented by the bold solid lines. Compared to the linear SEM, these two sets of approximate CEs for the SPM do not fall on top of each other. Here, the parametric bootstrap CE tends to be slightly tighter across the range of both types of emotions compared to the Wald-type CE. The bootstrap CE is therefore more likely to reject the test of linearity compared to the Wald-type CE.

Recall that linearity of the unknown latent function may be statistically evaluated by using the CEs to test the null hypothesis $H_0 : E[\eta_2|\eta_1] = \alpha_0 + \beta_0\eta_1$, where α_0 and β_0 can take on any real value. When a linear function is bounded within the CEs, this null hypothesis cannot be rejected. Conversely, when no linear function is bounded within the CEs, nonlinearity of the latent regression is suggested. From the left bottom plot of Figure 2, the 90% CEs for the relationship between positive emotions and heuristic processing can contain several linear functions. For instance, the linear function $E[\eta_2|\eta_1] = 0.2 + 0.1\eta_1$ lies within the boundaries of both types of CEs (not graphed). Therefore, there is no strong evidence in support of a nonlinear relationship between positive emotions heuristic processing.

The 90% Wald-type CE for the effect of negative emotions and heuristic processing was found to contain a line, which is depicted as a gray solid fine line in the right bottom plot of Figure 2. However, no such line was found to be contained within the analogous 90% parametric bootstrap CE. This noted discrepancy is due to the parametric bootstrap CE being slightly more efficient compared to the Wald-type CE. Taken together, these observations provide some evidence that the effect of negative emotions on heuristic processing is nonlinear. As the SPM and CEs here are used as exploratory devices, future studies could be designed to confirm the nonlinear form linking negative emotions to heuristic processing.

We have demonstrated that the SPM is a generalization of the linear SEM and has the advantage of relaxing the assumption of linearity to recover potential nonlinearity between latent variables. The SPM also has the added advantage over parametric approaches to modeling nonlinear latent relationships by making minimal distributional assumptions and avoiding a priori specification of the functional form between latent predictor and latent outcome. As the flexibility of the SPM reduces bias at the cost of efficiency, as illustrated in the CIs and CEs between the linear SEM of Figure 1 and the SPM of Figure 2, the SPM is better suited for exploratory research. The higher uncertainty in exploratory research is built into the SPM in terms of inefficiency in estimation as reflected in larger sampling variability.

Visualization Tool

Applying the SPM as an exploratory modeling device, especially generating plots of the recovered function and its CBs, requires considerable post-processing of results. To promote accessibility of this model to applied researchers, we introduce a new utility that builds on two existing tools written in R (R Core Team, 2013) that generates plots of (a) the aggregate latent function, (b) the marginal mixture densities for the latent variables, (c) the bivariate contour plot, and (d) the class probabilities across the range of the latent predictor (Pek et al., 2009). The new utility repackages these existing tools as an interactive web application using the R package "shiny" (RStudio, Incorporation, 2014) and is enhanced with the inclusion of newly developed Wald-type and parametric bootstrap CIs and CEs (Pek et al., 2011; Pek & Chalmers, 2015). The utility can generate plots of the four types of CBs: (a) Wald-type CIs, (b) parametric bootstrap CIs, (c) Wald-type CEs, and (d) parametric bootstrap CEs. Additionally, point-wise Wald-type and parametric bootstrap CIs about predicted latent outcome values $E[\eta_2|\eta_1]$ could be computed with user-specified conditional values on the latent predictor η_1. This utility also includes an implementation of the line finding algorithm, which could be run to diagnose nonlinearity of the recovered latent function when CEs are computed. The web application has been written to read Mplus (Muthén & Muthén, 2011) output files automatically so as to seamlessly generate CIs and CEs. As with the older utilities, users can continue to manually input estimates from other software such as OpenMx (Boker et al., 2011) to obtain plots other than the CIs and CEs. We provide instructions and some simulated examples on the use of this utility in the accompanying Online Appendix. Mplus output files for the empirical example relating emotions with heuristic processing are available online with the Online Appendix.

Summary and Conclusion

Research conducted in the educational, social, and behavioral sciences often employ latent variable modeling techniques. Although latent constructs may not necessarily be linearly related, this simplifying assumption is often made with little theoretical backing during the early stages of research. When the functional form between latent variables is unknown, the SPM is a useful exploratory approach to recover the unspecified latent relationship. A visualization of the recovered latent function with the addition of CIs or CEs enhances its description and interpretation. In particular, CIs and CEs communicate information regarding the precision of estimates and may be inverted to conduct informative

NHSTs. To promote the addition of this method to researchers' toolkit of exploratory latent variable methods, this article illustrates use of the SPM, including its CIs and CEs, and introduces a web application that automatically processes model results for ease of interpretation.

The SPM and especially its CEs are ideal for exploring whether the form of any unknown latent bivariate relationship is linear. Recall that to detect potential linearity, a line finding algorithm was developed as an implementation of the NHST of $H_0 : E[\eta_2|\eta_1] = \alpha_0 + \beta_0\eta_1$ (Pek & Chalmers, 2015). Users should be aware that this approach to evaluating nonlinearity is a graphical (or numerical) implementation, and results obtained from the search algorithm are not definitive but diagnostic at best. In conclusion, the semiparametric nature of these tools are best applied in an exploratory manner, and the providence of the web application as a computing resource should further encourage the exploration of potential nonlinearity among latent variables.

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Note

1. The slightly positive slope for the locally linear component at higher levels of negative emotions was not significantly different from zero.

Supplementary Material

The online appendices are available at http://jeb.sagepub.com/supplemental.

References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control*, *19*, 716–723. doi:10.1109/TAC.1974.1100705

Alloy, L. B., & Abramson, L. Y. (1979). Judgment of contingency in depressed and non-depressed students: Sadder but wiser? *Journal of Experimental Psychology: General*, *108*, 441–485. doi:10.1037/0096-3445.108.4.441

American Educational Research Association. (2006). Standards for reporting on empirical social science research in AERA publications. *Educational Researcher*, *35*, 33–40. doi:10.3102/0013189X035006033

American Psychological Association. (2010). *Publication manual of the American Psychological Association*. Washington, DC: Author.

Arminger, G., & Stein, P. (1997). Finite mixtures of covariance structure models with regressors: Loglikelihood function, minimum distance estimation, fit indices, and a complex example. *Sociological Methods & Research*, *26*, 148–182. doi:10.1177/0049124197026002002

Arminger, G., Stein, P., & Wittenberg, J. (1999). Mixtures of conditional mean-and covariance-structure models. *Psychometrika*, *64*, 475–494. doi:10.1007/BF02294568

Bauer, D. J. (2005). A semiparametric approach to modeling nonlinear relations among latent variables. *Structural Equation Modeling*, *12*, 513–535. doi:10.1207/s15328007sem1204_1

Bauer, D. J., Baldasaro, R. E., & Gottfredson, N. C. (2012). Diagnostic procedures for detecting nonlinear relationships between latent variables. *Structural Equation Modeling*, *19*, 157–177. doi:10.1080/10705511.2012.659612

Boker, S., Neale, M., Maes, H., Wilde, M., Spiegel, M., & Brick, T., … others (2011). OpenMx: An open source extended structural equation modeling framework. *Psychometrika*, *76*, 306–317. doi:10.1007/s11336-010-9200-6

Bollen, K. A. (1989). *Structural equation models with latent variables*. New York, NY: Wiley.

Cleveland, W. S. (1979). Robust locally weighted regression and smoothing scatterplots. *Journal of the American Statistical Association*, *74*, 829–836. doi:10.1080/01621459.1979.10481038

Cleveland, W. S., & Devlin, S. J. (1988). Locally weighted regression: An approach to regression analysis by local fitting. *Journal of the American Statistical Association*, *83*, 596–610. doi:10.1080/01621459.1988.10478639

Dolan, C. V., & van der Maas, H. L. (1998). Fitting multivariage normal finite mixtures subject to structural equation modeling. *Psychometrika*, *63*, 227–253. doi:10.1007/BF02294853

Efron, B., & Tibshirani, R. (1993). *An introduction to the bootstrap*. New York, NY: Chapman & Hall.

Jedidi, K., Jagpal, H. S., & DeSarbo, W. S. (1997a). Finite-mixture structural equation models for response-based segmentation and unobserved heterogeneity. *Marketing Science*, *16*, 39–59. doi:10.1287/mksc.16.1.39

Jedidi, K., Jagpal, H. S., & DeSarbo, W. S. (1997b). STEMM: A general finite mixture structural equation model. *Journal of Classification*, *14*, 23–50. doi:10.1007/s003579900002

Leemis, L. M., & McQueston, J. T. (2008). Univariate distribution relationships. *The American Statistician*, *62*, 45–53. doi:10.1198/000313008X270448

Maxwell, S. E., & Delaney, H. D. (2004). *Designing experiments and analyzing data: A model comparison perspective*. New York, NY: Psychology Press.

Meredith, W. (1993). Measurement invariance, factor analysis and factorial invariance. *Psychometrika*, *58*, 525–543. doi:10.1007/BF02294825

Muthén, B. O. (2001). Latent variable mixture modeling. In G. A. Marcoulides & R. E. Schumacker (Eds.), *New developments and techniques in structural equation modeling* (pp. 1–33). Mahwah, NJ: Lawrence Erlbaum.

Muthén, L. K., & Muthén, B. O. (2011). *Mplus user's guide* (6th ed.) [Computer software manual]. Los Angeles, CA: Author.

Neter, J., Kutner, M. H., Nachtsheim, C. J., & Wasserman, W. (1996). *Applied linear statistical models* (4th ed.). Chicago, IL: Irwin.

Nylund, K. L., Asparouhov, T., & Muthén, B. O. (2007). Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation study. *Structural Equation Modeling*, *14*, 535–569. doi:10.1080/10705510701575396

Pek, J., & Chalmers, R. P. (2015). Diagnosing nonlinearity with confidence envelopes for a semiparametric approach to modeling bivariate nonlinear relations among latent variables. *Structural Equation Modeling*, *22*, 288–293. doi:10.1080/10705511.2014.937790

Pek, J., Losardo, D., & Bauer, D. J. (2011). Confidence intervals for a semiparametric approach to modeling nonlinear relations among latent variables. *Structural Equation Modeling*, *18*, 537–553. doi:10.1080/10705511.2011.607072

Pek, J., Sterba, S. K., Kok, B. E., & Bauer, D. J. (2009). Estimating and visualizing nonlinear relations among latent variables: A semiparametric approach. *Multivariate Behavioral Research*, *44*, 407–436. doi:10.1080/00273170903103290

Perneger, T. V. (1998). What's wrong with Bonferroni adjustments. *British Medical Journal*, *316*, 1236–1238. doi:10.1136/bmj.316.7139.1236

R Core Team. (2013). *R: A language and environment for statistical computing* [Computer software manual]. Vienna, Austria: Author. Retrieved from http://www.R-project.org/

Raykov, T., & Marcoulides, G. A. (2004). Using the delta method for approximate interval estimation of parameter functions in SEM. *Structural Equation Modeling*, *11*, 621–637. doi:10.1207/s15328007sem1104 7

RStudio, Incorporation. (2014). *Shiny: Web application framework for R* [Computer software manual]. Retrieved from http://CRAN.R-project.org/package=shiny

Scheffé, H. (1953). A method for judging all contrasts in the analysis of variance. *Biometrika*, *40*, 87–110. doi:10.1093/biomet/40.1-2.87

Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics*, *6*, 461–464. doi:10.1214/aos/1176344136

Schwarz, N. (2000). Emotion, cognition, and decision making. *Cognition & Emotion*, *14*, 433–440. doi:10.1080/026999300402745

Schwarz, N., & Clore, G. L. (1996). Feelings and phenomenal experiences. In E. T. Higgins & A. Kruglanski (Eds.), *Social psychology: Handbook of basic principles* (pp. 385–407). New York, NY: Guilford.

Thissen, D., & Wainer, H. (1990). Confidence envelopes for item response theory. *Journal of Educational and Behavioral Statistics*, *15*, 113–128. doi:10.3102/10769986015002113

Titterington, D. M., Smith, A. F., & Makov, U. E. (1985). *Statistical analysis of finite mixture distributions*. Chichester, England: Wiley.

Tukey, J. W. (1980). We need both exploratory and confirmatory. *The American Statistician*, *34*, 23–25. doi:10.1080/00031305.1980.10482706

Wald, A. (1943). Tests of statistical hypotheses concerning several parameters when the number of observations is large. *Transactions of the American Mathematical Society*, *54*, 426–482. Retrieved from http://www.jstor.org/stable/1990256

Wilkinson, L., & the Task Force on Statistical Inference. (1999). Statistical methods in psychology journals: Guidelines and explanations. *American Psychologist*, *54*, 594. doi:10.1037/0003-066X.54.8.594

Working, H., & Hotelling, H. (1929). Applications of the theory of error to the interpretation of trends. *Journal of the American Statistical Association*, *24*, 73–85. doi:10.1080/01621459.1929.10506274

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